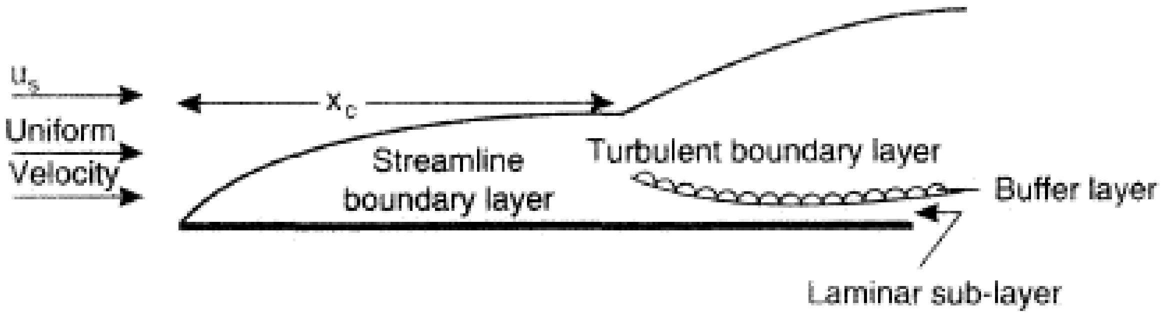
HW

Calculate the thickness of the boundary layer at a distance of 75 mm from the leading edge of a plane surface over which water is flowing at a rate of 3 m/s. Assume that the flow in the boundary layer is ***streamline*** and that the velocity u of the fluid at a distance y from the surface can be represented by the relation $u = a + by + cy^2 + dy^3$, where the coefficients a, b, c, and d are independent of y. The viscosity of water is 1 mN s/m$^2$.




## Solution

At a distance y from the surface: $u = a + by + cy^2 + dy^3$.

When y = 0, u = 0, and hence a = 0.

The shear stress within the fluid: $R_o = -\mu(\partial u/\partial y)_{y=0}$ and since $\partial u/\partial y$ is constant for small values of y, $(\partial^2 u/\partial y^2)_{y=0} = 0$. At the edge of the boundary layer, $y = \delta$ and $u = u_s$, the main stream velocity.

$\partial u/\partial y = 0$ and $u = by + cy^2 + dy^3$

$\partial u/\partial y = b + 2cy + 3dy^2$ and $\partial^2 u/\partial y^2 = 2c + 6dy$

When $y = 0$, $\partial^2 u/\partial y^2 = 0$, and hence $c = 0$.

When $y = \delta$, $u = b\,\delta + d\delta^3 = u_s$

and: $\partial u/\partial y = b + 3d\,\delta^2 = 0$

$\therefore b = 3\,d\,\delta^2$

$d = -u_s/2\,\delta^3$ and

$b = 3u_s/2\delta$

The velocity profile is given by, $u = (3u_s y/2\delta) - (u_s/2)(y/\delta)^3$ or: $u/u_s = 1.5y/\delta - 0.5\,(y/\delta)^3$

$$\frac{\delta}{x} = 4.64 \quad Re_x^{-1/2}$$

$$Re_x = \left(\frac{0.075 \times 3 \times 1000}{1 \times 10^{-3}}\right) = 225000$$

$\delta/x = 4.64 \times (225000)^{-1/5} = 0.00978$

$\delta = 0.00978 \times 0.075 = = 0.000734$ m or 0.734 mm

**Example (1)**

Water flows at a velocity of 1 m/s over a plane surface 0.6 m wide and 1 m long. Calculate the ***total drag force*** acting on the surface if the transition from streamline to turbulent flow in the boundary layer occurs when the Reynolds group $Re\ x_c = 10^5$.(Taking μ = 1 mN s/m$^2$ )

# Solution

μ = 1 mN s/m$^2$ = $10^{-3}$ Ns/m$^2$,

at the far end of the surface, $Re_x = (1 \text{ x } 1 \text{ x } 10^3)/10^{-3} = 10^6$

Mean value of $\left(\frac{\mathbf{R}}{\boldsymbol{\rho}\ \mathbf{u_s^2}}\right)$ from equation (24)

$$\left(\frac{\mathbf{R}}{\boldsymbol{\rho}\ \mathbf{u_s^2}}\right)_m = \mathbf{0.037\ Re_x^{-0.2}} + \mathbf{Re_x^{-1}}\ [\mathbf{0.646\ \boldsymbol{Re_{x_c}^{0.5}} - 0.037\ Re_{x_c}^{0.8}}]$$

$$= 0.037\times(10^5)^{-0.2} + (10^5)^{-1}\ [0.646(10^5)^{0.5} - 0.037(10^5)^{0.8}$$

= 0.002

Total drag force = $\frac{\mathbf{R}}{\boldsymbol{\rho}\ \mathbf{u_s^2}}\ (\boldsymbol{\rho}\ \mathbf{u_s^2}) \times$ (area of surface)

= (0.002 × 1000 × (1)$^2$ × (1 x 0.6)

= 1.2 N

Example 2

Calculate the thickness of the boundary layer at a distance of 150 mm from the leading edge of a surface over which oil, of viscosity 0.05 N s/m$^2$ and density 1000 kg/m$^3$ flows with a velocity of 0.3 m/s. What is the thickness of the boundary layer?

## Solution

$Re_x = (0.150 \times 0.3 \times 1000/0.05) = 900$

For streamline flow: $\frac{\delta}{x} = \frac{4.64}{Re_x^{0.5}}$ (from equation 8)

$= \frac{4.64}{(900)^{0.5}} = 0.1545$

Hence: $\delta$ = (0.1545 x 0.150) = 0.0232 m = 23.2 mm

Example 3

Calculate the thickness of the laminar sub-layer when benzene flows through a pipe 50 mm in diameter at **2 l/s**. What is the velocity of the benzene at the edge of the laminar sub-layer? Assume that fully developed flow exists within the pipe and that for benzene, $\rho$ = 870 kg/m$^3$ and μ = 0.7 mN s/m$^2$.

## Solution

The mass flowrate of benzene G = $v(m^3/s) \times \rho\ (kg/m^3)$ = $(2\times10^{-3} \times 870)$ =1.74 kg/s

$$[Re = \frac{\rho u d}{\mu} = \frac{Gd}{A\mu} = \frac{4G}{\mu \pi d}]$$

Thus: Reynolds number $= \dfrac{4G}{\mu(\pi D)} = \dfrac{4 \times 1.74}{0.7 \times 10^{-3}(\pi \times 0.050)} = 63{,}290$

From equation (29) : $\dfrac{\delta_b}{d} = 62\ Re^{-7/8}$ $\qquad \delta_b = \dfrac{(62\ x\ 0.050)}{(63.290)^{7/8}} = 1.95\times10^{-4}\ m = 0.195 mm$

The mean velocity = $\dfrac{G}{\rho\ A} = \dfrac{1.74}{870\times(\frac{\pi}{4}\times0.05^2)} = 1.018$ m/s

From equation(26) $\qquad \dfrac{u_b}{u_s} = 2.49\ Re^{-\frac{1}{8}}$

from which: $u_b = \dfrac{2.49 \times 1.018)}{63{,}290^{1/8}} = 0.637\ m/s$

# Basic Principle of Unit Operations

$$\text{Rate of transport} = \frac{\mathrm{d}(amount\ of\ property\ transferred)}{\mathrm{d}(time)} = \frac{Driving\ force}{Resistance}$$

| Process | Driving force | Resistance |
|---|---|---|
| Fluid Flow | Pressure | Viscosity and density of fluid |
| Heat transfer | Temperature | Conductivity of solid |
| Mass Transfer | Concentration | Diffusivity |

## Momentum, Heat and Mass Transfer

In most of the unit operations encountered in the chemical and process industries, one or more of the processes of momentum, heat, and mass transfer is involved. Thus, in the flow of a fluid under *adiabatic conditions* through a bed of granular particles, in the direction of flow a **pressure gradient** is formed. While a velocity gradient develops approximately perpendicularly to the direction of motion of fluid stream, **momentum transfer then takes place** between the elements of fluid which are moving at different velocities. If there is a temperature difference between the fluid and the pipe wall or the particles, **heat transfer will take place** as well, and the convective component of the heat transfer will be directly affected by the flow pattern of the fluid.

Here, then, is an example of a process of simultaneous **momentum and heat transfer** in which the same fundamental mechanism is affecting both processes. **Fractional distillation** and **gas absorption** are frequently carried out in a **packed column** in which the gas or vapour stream rises counter-currently to a liquid. The function of the packing in this case is to provide a large interfacial area between the phases and to promote turbulence within the fluids. In a very turbulent fluid, the rates of transfer per unit area of both **momentum and mass are high**; and as the **pressure drop rises** the rates of transfer of both **momentum and mass increase together**. In some cases, **momentum, heat, and mass transfer all occur simultaneously** as, for example, in a water-cooling tower, where transfer of sensible heat and evaporation both take place from the surface of the water droplets.

When a fluid is flowing under **streamline conditions** over a surface, a forward component of velocity is superimposed on the random distribution of velocities of the molecules, and movement at right angles to the surface occurs only as a result of the ***random motion of the molecules***. Thus if two adjacent layers of fluid are moving at different velocities, there will be a tendency for the faster moving layer to be retarded and the slower moving layer to be accelerated by influence of the continuous passage of molecules in each direction, There will therefore be a net transfer of momentum from the fast to the slow moving stream. Similarly, the molecular motion will tend to reduce any temperature gradient or any concentration gradient if the fluid consists of a mixture of two or more components. A the boundary the effects of the molecular transfer are balanced by the drag forces at the surface.

## Fluid flow types:

- **Laminar flow :**

Laminar flow generally happens when dealing with small pipes and low flow velocities. Laminar flow can be regarded as a series of liquid cylinders in the pipe, where the deepest parts flow the fastest, and the cylinder touching the pipe isn't moving at all.

- **Turbulent flow:**

In turbulent flow vortices, eddies and waves make the flow random. Turbulent flow happens in general at high flow rates and with larger pipes.

- **Transitional flow:**

Transitional flow is a mixture of laminar and turbulent flow, with turbulence in the center of the pipe, and laminar flow near the edges.

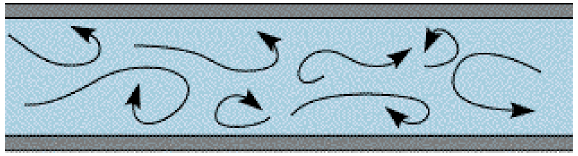
When a fluid flows under **turbulent conditions over a surface**, the flow can be divided into three regions:

❖ At the surface, **the laminar sub-layer**, in which the only motion at right angles to the surface is due to **molecular diffusion**. The resistance to transfer of momentum, heat and mass concentrated in this region and can be reduced by increasing the Reynolds number.

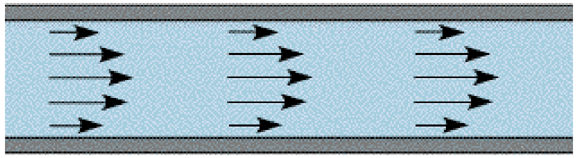
❖ Next, **the buffer layer**, in which molecular diffusion and eddy motion are of comparable magnitude (50% sub-layer + 50% turbulent core).

❖ Finally, over the greater part of the fluid, **the turbulent region** in which eddy motion is large compared with molecular diffusion.

Turbulent



Laminar



**Laminar Flow**


**Turbulent Flow**

## Momentum Transfer

The shear stress ($R_y$) in a fluid is proportional to the velocity gradient and to the viscosity, When the flow characteristics of the fluid are Newtonian.

$$R_y = \frac{-\mu}{\rho}\frac{d(\rho u_x)}{dy}$$

## Heat Transfer

The heat transferred per unit time through a unit area at a distance ( y ) from the surface is given by :

$$q_y = -K\frac{d\theta}{dy} = q_y = -\frac{K}{\rho C_p}\frac{d(\rho C_p \theta)}{dy}$$

**Where:**

Cp : is the specific heat of the fluid at constant pressure (J/kg . K).

θ : is the temperature , (K)

k : is the thermal conductivity , (W/m .K)

($\frac{K}{\rho c_p}$): is the thermal diffusivity , ($m^2$/s)

# Mass Transfer

The rate of diffusion of a constituent A in a mixture is proportion to its concentration gradient .

$$N_A = -D\frac{dC_A}{dy}$$

Where :

$N_A$ : is the molar rate of diffusion of constituent A per unit area , (Kmol / $m^2$ .s)

$C_A$ :is the molar concentration of constituent A , (Kmol/$m^3$)

D : is the mass diffusivity , ($m^2$/sec.)